

Investigation of Testing System for Solid Propellant Rocket Motor Thrust Measurements Using Matlab Mathematical Modelling Techniques

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ABSTRACT

Rocket propellant has been identified as a component that played an important role in the development of rockets aerospace technology even new earlier of auto space discovery. The need to determine the mathematical modelling of locally constructed propeller gas rocket engine motor is newer in technology and help enhancing performance in the development of propeller gas rocket engine. There need to adopt mathematical modelling technique of locally constructed propeller gas rocket engine motor which enhancing performance in the development of propeller gas rocket engine. Design technique describes solid fuel rocket engine motor testing system for its thrust measuring capabilities. The design process consists of two parts. In first part the mathematic modelling technique will be described for theme chemical section, in second-electrical. This will help to explain versatility and capabilities for coupling of the parametric design process for measurement systems design using software supported modelling tools. In order to determine the potential utility load, module design and the stability of the system in next chapter model of the system is compared with actual measurement system prototype during experimental testing and evaluation. The error between modelled and real systems is measured and discussed. The final experiment is discussed where rocket motor was tested. Modelled thrust characteristics were compared with measured data for the final discussion and conclusions.

KEYWORDS

Rocket engine motor, thrust, measuring system, modelling, tests and results

I. INTRODUCTION

One of the major challenges in the rocket science is to maintain nominal rocket motor performance during the lift off and the rest of the flight. This concludes to problems where internal ballistics is the one of the primarily part of the equation. There are four basic key elements where one can determine the rocket engine motor performance characteristics: Thrust, pressure, temperature and acoustics. This paper is based on first and most important component: thrust measurements of the rocket motor. This problem employs the requirement of the analysis for the two fundamental parts in thrust measurement systems-relationship of the mechanical and electrical hardware coupling. Many research groups which

are dedicated to thrust systems design using relatively the same methods for propulsion systems thrust measurements. Most common thrust measuring systems is hydraulic, piezoelectric. Or load cell base. Hydraulic systems are losing its interest in thrust measurements because of more reliable electronic measurement systems. Most robust and versatile is load cell based measuring systems because of their capability to use the same load cell for different ranges of trust measurements. In this paper one describes a load cell with two working points for different thrust rocket motors testing. The solid rocket motor testing stand consists of testing stand structure, thrust sensor, amplifier and analogue–digital converter, recording device. The testing stand structure has to maintain rapidly changing thrust force based on X direction (in this particular system setup) and at the same time to be able suppress motor displacement along $-X$ direction but let motor slide as free as possible in $+X$ direction. In other case adverse oscillations may occur (Mason et.al 2019). This set up ensures reliable thrust measurements during experimental testing. In the next chapter the behaviour of the testing stand structure will be discussed more.

The thrust measurement sensor (load cell) consists of machined metal part where strain gauge array is attached in Wheatstone bridge setup. In this particular setup four strain gauge sensors were used in pairs by active and balancing arrangement. Specifically, for this system load cell was designed for two measuring ranges: 0-10kN and 0-15kN. This is crucial to use suitable load cell working point where measuring range is bigger than designed maximum motor thrust because of possibility of unstable fuel grain burn and probability of thrust spikes formation which can cause damaging of the measuring system (Marble et.al 2023). The amplifier was designed using operational amplifiers in the instrumental amplifier setup. The calculations and modelling was performed based on load cell mechanical characteristics which were derived from equations. As data recording unit one was used computer with Bruel & Kjaer "Pulse" software.

The aim of this paper is to propose Investigation of Testing System for Solid Propellant Rocket Motor Thrust Measurements Using Matlab Mathematical Modelling Techniques methods for load cell based thrust measurement systems which can be designed parametrically with ease of capability to customize and adapt working ranges of thrust measurement for solid rocket motor

A. Applications

Currently, rockets have applications both in space and on earth. They are used for research works at high altitudes, in assisting heavy aircrafts during take-offs, in SOS application to invite rescue teams to ships that are stranded offshore like distressed ships. Rockets also assist pilots' ejection during aircraft emergencies and for entertainments with fireworks. Rockets have been used to launch satellites and enhance communications. Probably, the use of rockets for military purposes, space exploration and steering of vehicles in space are the best known in rocket application.

B. Statement of the Problem

- There are some problem in data related to solid rocket propellant is too less in order to serve as a reference guide of funds in this areas on the use of gas propeller rocket engines showing detailed

- mathematical model for efficiency, durability
- And for National budget reduction cost in our military supply base.
- The other hand accurate data precision on the burning rate test and conditioned

Usage of our raw materials for manufacturing purposes its limited even not harnessed So, there are a few expected works to do on based engine rocket solid propellant developed locally in the military defense base will be used.

C. *Aims and Objectives of Present Research*

The specific research objectives of the study are:

- Sets running the constructed modernized Gas Rocket Engine propeller using locally available Used in Military Defence Kaduna State Nigeria
- Shows it mathematical modelling
- Validation of the empirical kinetic equations modelling, and Simulation

D. *Research Gap*

Previous studies on rocket engine design and performance have largely focused on advanced liquid and solid propellant systems developed in technologically advanced countries, with limited attention given to the mathematical modelling and simulation of simple gas-propelled rocket engines using locally available fuels and materials in Nigeria. Most existing studies emphasize theoretical analyses or computational simulations without validating the results through experimental prototype testing. In addition, many available studies rely on imported materials and sophisticated manufacturing processes that are not readily accessible for indigenous research and defence applications. Consequently, there is limited evidence regarding the feasibility of developing a locally fabricated prototype rocket engine capable of producing performance characteristics comparable with analytical predictions.

II. **ENGINE ROCKET MOTOR**

The engine rocket motor is shown in the Fig.1 and consists of seven main parts: 1.Igniter, 2.Pressure cover, 3.Nozzle, 4.Fuel grain, 5. Grain casing and thermal protection layer, 6.O-rings and gaskets, 7. Head. The designed characteristics of the rocket engine motor are described



F

Figure 1: Solid propellant rocket engine motor components

Table 1: The Solid propellant rocket engine motor characteristics

Properties	Values	Units
Nominal thrust	11999.00	N
Maximum thrust	12999.00	N
Average thrust	11000.00	N
Burn time	3.6.00	s
Propellant mass	17.90	kg
Motor mass	32.50	kg
Total impulse	33000.00	Ns
Specific impulse	188.00	s

Rocket motor produces thrust when burned propellant escapes from throat of the rockets nozzle. It is crucial that mass flow rate Eq. (a) would be stable at all period of motor operation cycle until sliver zone is reached because otherwise the oscillations in thrust graph will occur and this could lead in burn instability and increased measurement error possibility:

$$\dot{m} = \rho A_s b_r, \quad (a)$$

Where: ρ – density of the propellant (kg/m³), A_s – surface area of the propellant grain (m²), b_r – burn rate (kg/s).

In this particular design the neutral star type (Kamran et.al. 2025) of propellant grain was used. Which means that surface area of the burning propellant is constant despite time increment until sliver is reached. To determine desired rocket motor thrust characteristics (Jenkins et.al. 2023) it is crucial to choose right cross section geometry of the propellant grain and establish desired area of burning surface because rocket motor thrust is directly related with fuel grain area A_s Eq.

$$\int A_s dt = \text{const} \text{ (m}^2\text{)}. \quad (b)$$



Figure 2: the Propellant grain during burning

The equations below describe how thrust is produced during burn Aqsa. (b), (c) and (f) of propellant and derivation of total impulse Eq. (d) and specific impulse Eq. (e) of the rocket engine motor:

$$F = \dot{m} V_e + (p_e - p_0) A_e \text{ (N)}, \quad (c)$$

Where: $\dot{m}$ = mass flow, V_e =exit velocity, p_e =exit pressure, p_0 =atmospheric pressure, A_e = exit area, F = thrust:

$$+ (p_e - p_0) A_e \quad / \overline{MS} \text{ eqe } \dot{m}$$

$$F = \dot{m} V_{eq} \text{ [N]}, \quad (e)$$

$$I = F\Delta t = \int F dt = \int m \dot{V}_{eq} dt = m V_{eq} \text{ (N}\cdot\text{s)}, \quad (f)$$

$$Sip = f/m_{go} - V_{eq}/g_0 - F/m_{go} \quad (g)$$

Where: g_0 –gravitational acceleration the surface of the earth, m – mass:

$$F = m \dot{V}_e + (p_e - p_0) A_e \text{ (N)}. \quad (h)$$

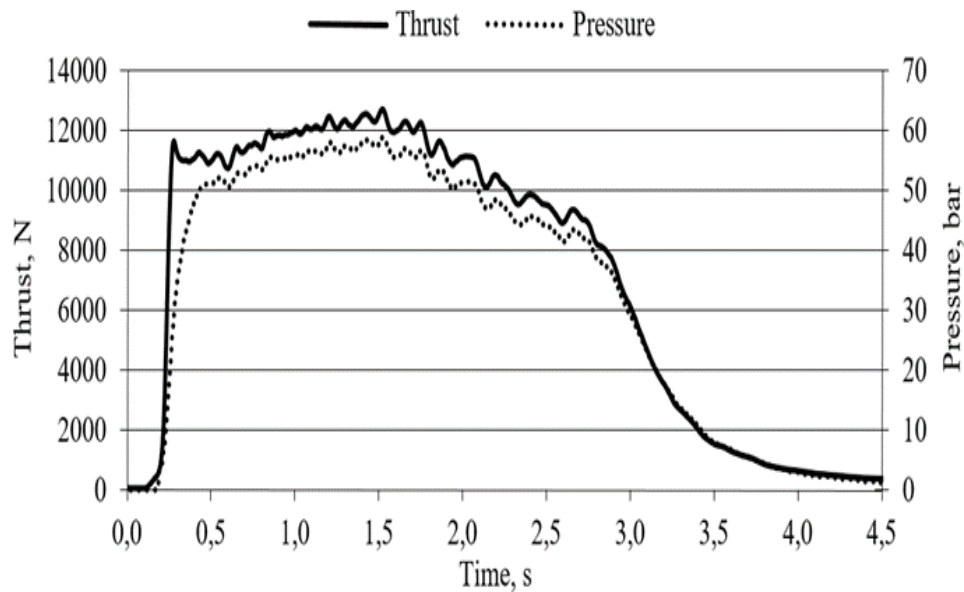


Figure 3: Modelling results of the rocket motor

After nozzle is added in to system equations became:

$$A p_t \sqrt{\frac{\gamma + 1}{\gamma - 1}} \frac{1}{R} \frac{2(\gamma - 1)(\gamma + 1)}{2} \quad (I)$$

$$\frac{p_e}{p_s} = \left(1 + \frac{\gamma}{2} \frac{m^2}{2}\right)^{-\frac{\gamma}{\gamma - 1}} \quad (j)$$

$$V = M C \sqrt{\gamma R T_e} \quad (K)$$

$$\frac{A_e}{A} = 2^{\frac{\gamma + 1}{2(\gamma - 1)}}, \gamma - 1_{x2y2} \left(\frac{\gamma - 1}{e}\right) y^{\gamma - 1} / m^2 \quad (L)$$

Where: A^* – throat area, p_t – throat pressure, T_t – throat temperature, γ – specific heat ratio, R – gas constant, M_e – Mach number at nozzle exit.

The equations were employed into mathematic model using Mat labs software. In Fig.3.0 one can see output graph of the designed rocket motor thrust and pressure curves.

III. LOAD CELL

Based on the modeling results the load cell was designed for working point $bb_1 = 0.75-10\text{kN}$ and working point $bb_2 = 0.5-15\text{kN}$ thrust measurement ranges. For load cell model D16T1 aluminum alloy was used $E = 75\text{ GPa}$ (Korobow et.al. 2010). It is recommended that working point bb should be in range of $0.5B \leq bb \leq B$. In Fig. 4 one can see load cell geometry with parameters. It must be noted that load cell thickness parameter in this geometry is not visible. This parameter is noted as it was recommended for the load cell thickness described as $B/8 \leq t \leq B/2$.

For the diameter parameter it is recommended that D should be in range of $B/3 \leq D \leq 2B/3$ and its center e be at $H/2 = B/2$. For H parameterization it is recommended to use values same as B . Next set of equations describe the behavior of the load cell and its mechanical properties Eq. (m) where axial stress is described, Eq. (n) bending stress, Eq. (o) stress to the sensors active zone, Eq. (o) maximum stress and in Eqs. (p) And (q) bending coefficient:

$$\sigma_a = \frac{-p}{wt} \left(\frac{N}{\text{mm}^2} \right) \quad (\text{m})$$

$$\sigma_b = \frac{-6pg}{wtc} \left(\frac{N}{\text{mm}^2} \right) \quad (\text{n})$$

$$\sigma_{\text{sensor}} = \sigma_a + \sigma_b \left(\frac{N}{\text{mm}^2} \right) \quad (\text{o})$$

$$\sigma_{\text{Max}} = -(\sigma_a + \sigma_b)K \left(\frac{N}{\text{mm}^2} \right) \quad (\text{P})$$

$$\text{Were, } k_1 = \frac{D+C}{2} = \frac{K}{2} = \frac{K}{C} = \frac{2}{\ln} \left(\frac{k+1}{k^2-1} \right)^2 = \frac{11}{3KK^2-1} \quad (\text{q})$$

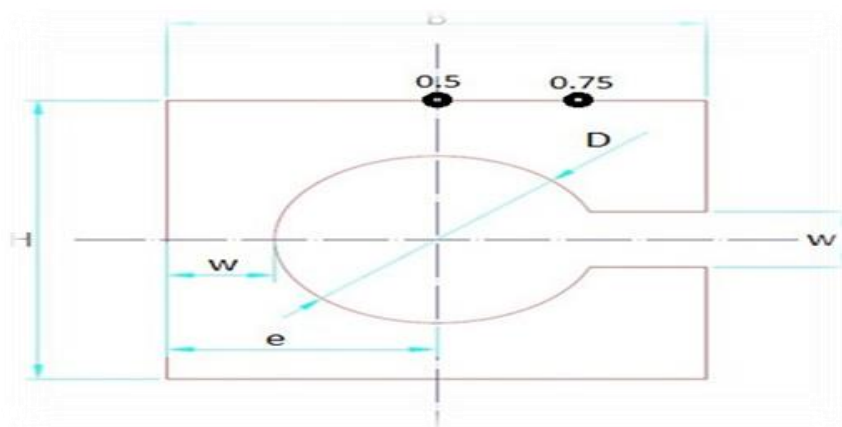


Figure 4: geometry of a Load cell

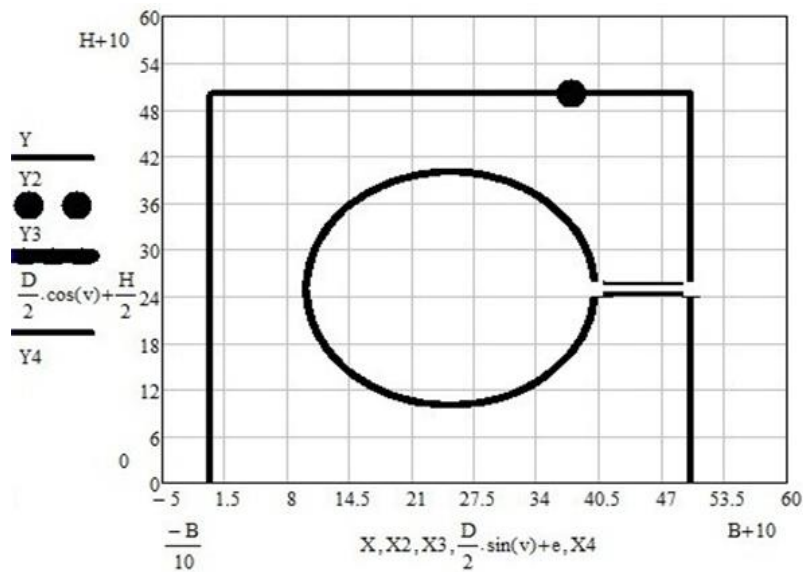


Figure 5: Load cell mechanical model geometry results graph

The strain of the load cell sensor point is calculated to check structural integrity of the whole system. It's recommended that the ε_{bend} should be in the range of $0 < \varepsilon_{bend} < 2000e-06$. It was found that best results are achieved when ε_{bend} is approximately in $1500e-06$ range. In this particular case the ε_{bend} was $1547e-06$. In Fig.5.0 one shows the modelled load cell for the particular case.

To calculate the elongation of the load cell at the sensor point for the design conditions of the 10kN and 15kN load points Eq. (m), Eq. (n) and Eq. (o) is used. This equation help sto-check if sensor of the load cell is capable to measure projected range of force. Typically, the ten zo sensor is capable to register measurements at max 1% or 1.8° of its bend. In this particular case the bend of 0.154% or 0.278° was reached. The sensor was SGD-7/350-DY43 (Gillette et. al., 2015). This data is crucial for designing robust instrumental amplifier

$$\Delta l = (B \cdot \varepsilon_{bend}) \rightarrow \Delta H = \Delta l + B \quad (\text{mm}) \quad (r)$$

$$\Delta l_{\%} = \Delta H - 100 / B \rightarrow \Delta l_{\%} = \Delta l_{\%} - 100(\%),$$

$$\Delta l_{\phi} = \frac{180}{\pi} \cdot \Delta l_{\%} (^{\circ})$$

100

IV. INSTRUMENTAL AMPLIFIER

Designing instrumental amplifier is crucial to calculate mechanical input data property, because one can deviate very aggressively when calibrating such system. Based on this particular case used strain gage one has found that k – the bending coefficient was 2.06. This coefficient may differ if another type of strain gages are used so careful study of

datasheet is recommended. For the amplifier four gages was used. Two was in active mode and two in passive for balancing purposes of the Wheatstone bridge Fig. 6.0

The input voltage for the amplifier was 5V. In Eq.(r) one is show the method for calculating the output voltage form the maximally unbalanced bridge (10Kn and 15kN) and how many times this output must be amplified (if output voltage is 5V) to be able registering and recording output signal Eq.(r). In this case one was found that out put voltage is 0.00796V and requirements for the amplifier are 627.455 times amplification of the input signal from the bridge:

$$G = 5 / V_{out} \quad (\text{times}).$$

For the sake of simplicity, instrumental amplifier was designed using same R_{in} values for all operational amplifiers (Fig. 7.0). R_{in} was selected for 10k Ω value and R_f was calculated based on amplifier requirements

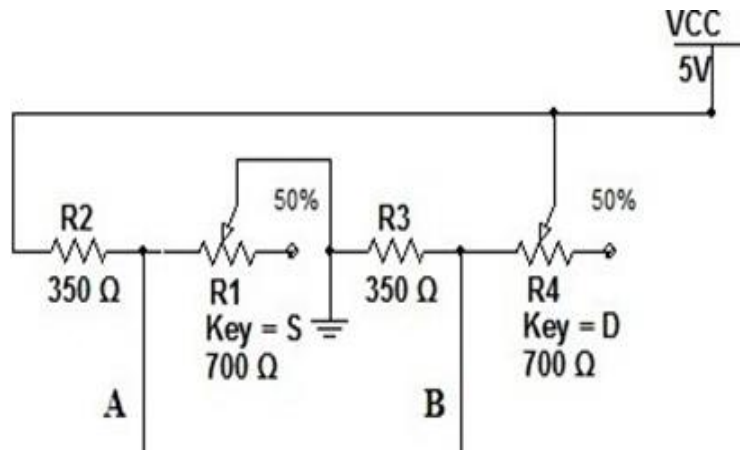


Figure 6: Wheatstone bridge

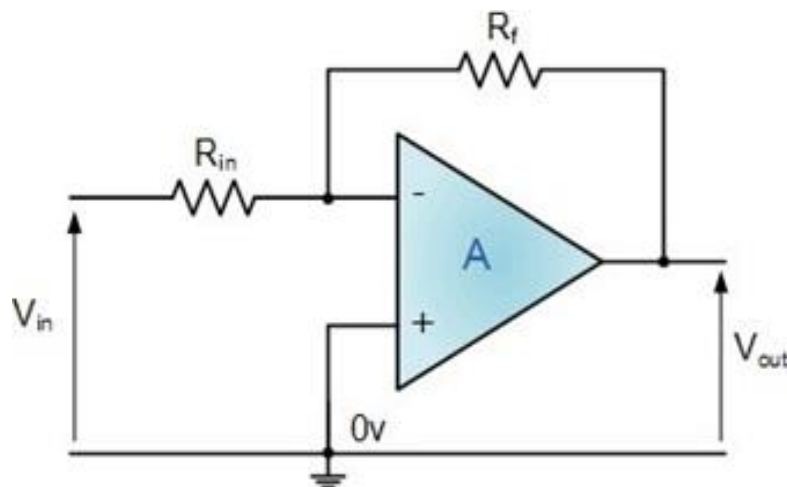


Figure 7: Operational amplifier

The first stage of amplification is done at buffer level (Fig.8.0). Since R_{1buff} and R_{3buff} is the R_{in} which are constant the G_1 is calculated using dependence of G_{spread} which is the quotient of the requirement of the amplification G divided by product of number operational amplifiers used in the system.

In this particular case the divisor for G_{spread} was 33 because one has used 3 operational amplifiers. So for the buffer stage G_1 was double the G_{spread} because 2 amplifiers in buffer stage were used. In this case G_1 was 46.478 and G_2 was 13.5 times of amplification. The values for R_f where in this case it was noted as R_{2buff} Eq. (s). One was found that

$$R_{2buff} = 430 \, \Omega:$$

$$R_{2buff} = (R_{1buff} + R_{3buff}) / (G_1 - 1) \, (\Omega) \quad (s)$$

$$R_{2buff} = (R_{1buff} + R_{3buff}) / (G_1 - 1) \, (\Omega) \quad (s)$$

These con-damplification stage (Fig.9.0) have two resistors R_{5amp} and R_{8amp} .

Because they the same it can be described as $R_{8amp}/R_{7amp} = R_{5amp}/R_{4amp}$. So to find R_{4amp} one has use Eq. (t):

$$R_{4amp} = R_{5amp} / G_2 \quad (t)$$

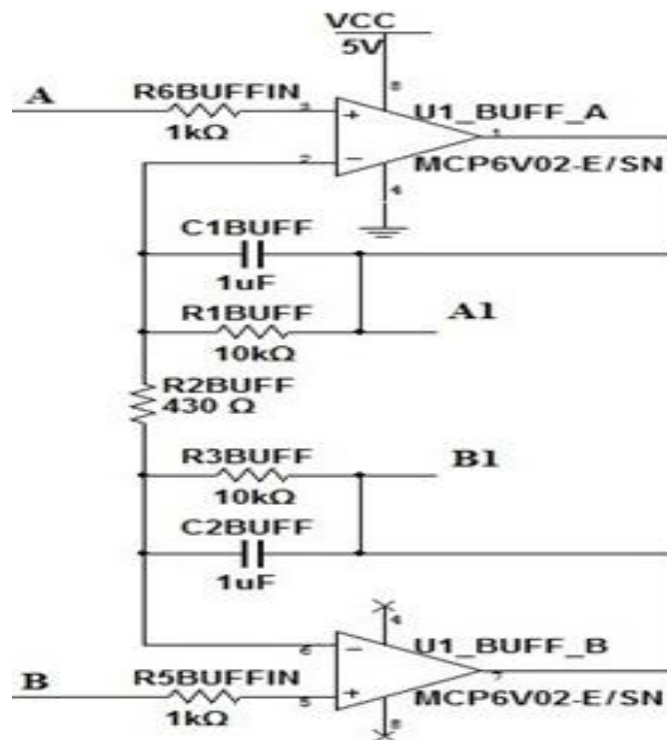


Figure 8: Buffer stage of instrumental amplifier

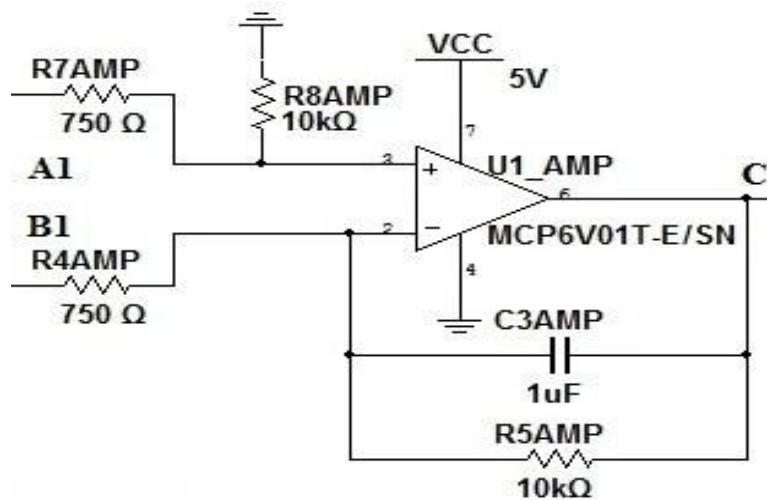


Figure 9: Amplification stage of instrumental amplifier

One must be noted that resistor values obtained from calculations not always those described can be used in reality because of resistor manufacturers using standard resistor values. In this case one must pick closest available resistor with smallest value deviation from calculated. By assessing this fact additional check is made to determine the error of the ideal and real modelled systems Eqs. (u), (v) and (w). One must be noted that R_2 and R_4 is resistor values which are closest by value to R_{2buff} and R_{4amp} resistor values which can be found in the market. V_{out} determine the output of volts in the real model. In this case one has found that V_{out} is equal to 5.048V. Considering that V_{out} is higher than power supply voltage one must be included in this surplus of voltage as error:

$$G_{out} = (1 + R_{1buff} + R_{3buff}/R_{2buff} \times R_{somp}/R_{4amp}) \text{ (times)} \quad (U)$$

$$G_{out} = (1 + R_{1buff} + R_{3buff}/R_2 \times R_{5amp}/R_4) \text{ (times)}$$

$$G_{out} = (1 + R_{1buff} + R_{3buff}/R_2 \times R_{5amp}/R_4) \text{ (times)}$$

$$V_{out} = V_{out} + (1 + R_{1buff} + R_{3buff}/R_{2buff} \times R_{somp}/R_{4amp} \times IVL) \quad (V)$$

$$V_{out} = V_{out} + (1 + 1 + R_{1buff} + R_{3buff}/R_{2buff} \times R_{somp}/R_{4amp} \times IVL) \quad (W)$$

$$V_{out} = V_{out} + (1 + R_{1buff} + R_{3buff}/R_{2buff} \times R_{somp}/R_{4amp} \times IVL)$$

Using the model which was described above analysis was made to determine system step response and settling time. First test was performed to check systems step response. Step was set for the maximum thrust of the rocket motor which was 15kN. In Fig.10.0 one can see results of the step response.

It was found that system error became minimal after 90.877 ms when ideal power supply was 5 V and fully unbalanced bridge produced 7.905 mV. For the settling case one was found that system fully settles after 56.907 ms when load cell is loaded 0 N.

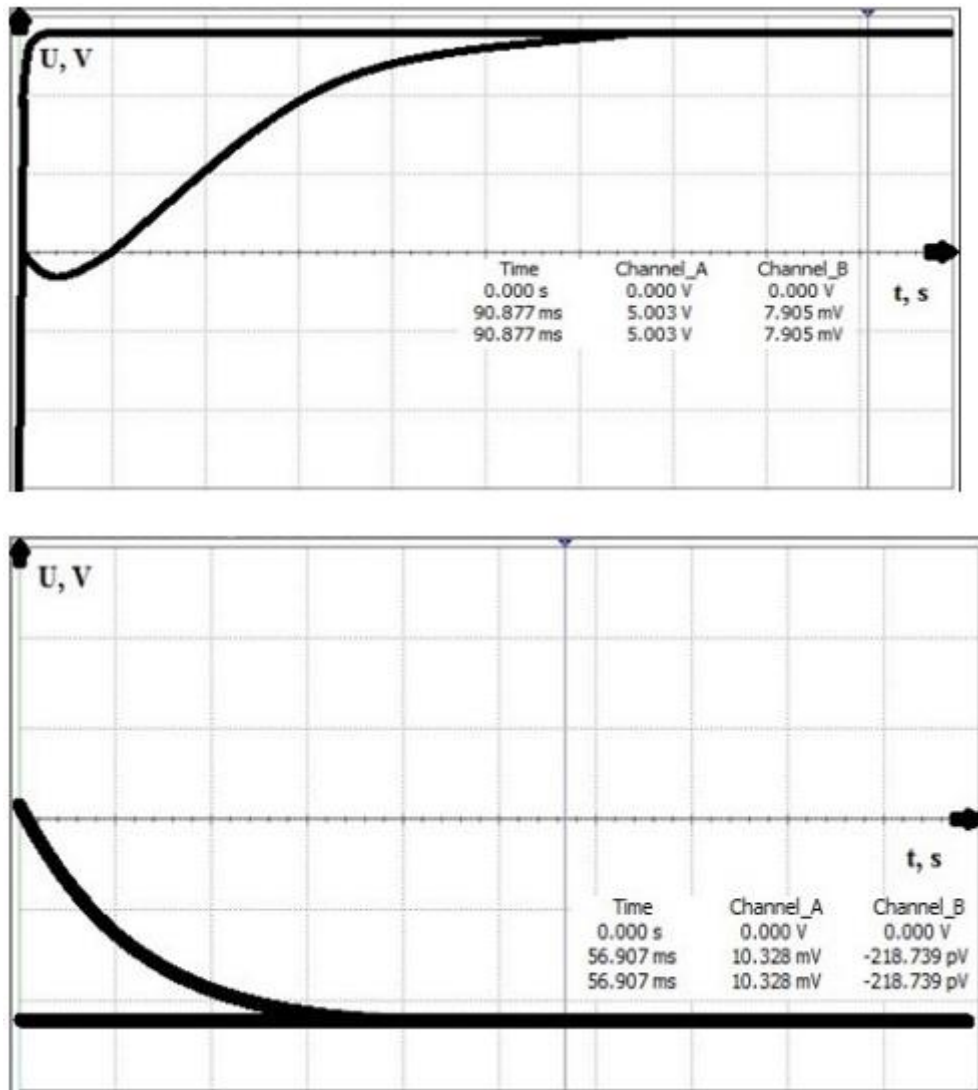


Figure 10: System reaction to step response and settling time

After comparing ideal and modeled systems it was found out that amplification error was 0.149%. Result was derived from equations below Eq.(x) and Eq. (y):

$$L_{ideal} = 20 \log (G_{out}) \text{ [dB]}, \quad (x)$$

$$L_{model} = 20 \log (G_{outt}) \text{ [dB]}. \quad (y)$$

V. EXPERIMENTAL TESTING

System was calibrated with high precision dynamometer where model results was compared with produced thrust measurement system based on modelling. in Fig.11.0 and in Table 2.0 one can see calibration results where load was applied to cover whole measurement range of the load cell. It must be noted that pressing system which was used for the calibration was capable to load for maximum of 11 kN where load cell was designed for maximum 15 kN. However, load cell was designed to work at two measurement ranges which was 10 kN and 15 kN.

Table 2: Results of Calibration

Load increment number	Dynamometer values, N	Model results, V	Reals ys tem results, V	Error, %
1.0	1000.0	0.500	0.509	1.800
2.0	2000.0	1.000	1.014	1.400
3.0	3000.0	1.500	1.515	1.000
4.0	4000.0	2.000	2.015	0.750
5.0	5000.0	2.500	2.519	0.760
6.0	6000.0	3.000	3.018	0.600
7.0	7000.0	3.500	3.52	0.571
8.0	8000.0	4.000	4.02	0.500
9.0	9000.0	4.500	4.517	0.377
10.0	10000.0	5.003	5.019	0.319

By limitation of testing hardware 1 of 2 possible working points was calibrated. After calibration one was found that measuring error was relatively small and was in range from 0.320% to 1.8%. Assuming that this system was designed for rocket motor which average thrust is 10kN one can consider that real error should not exceed 0.56 % in rocket burning time until sliver is reached

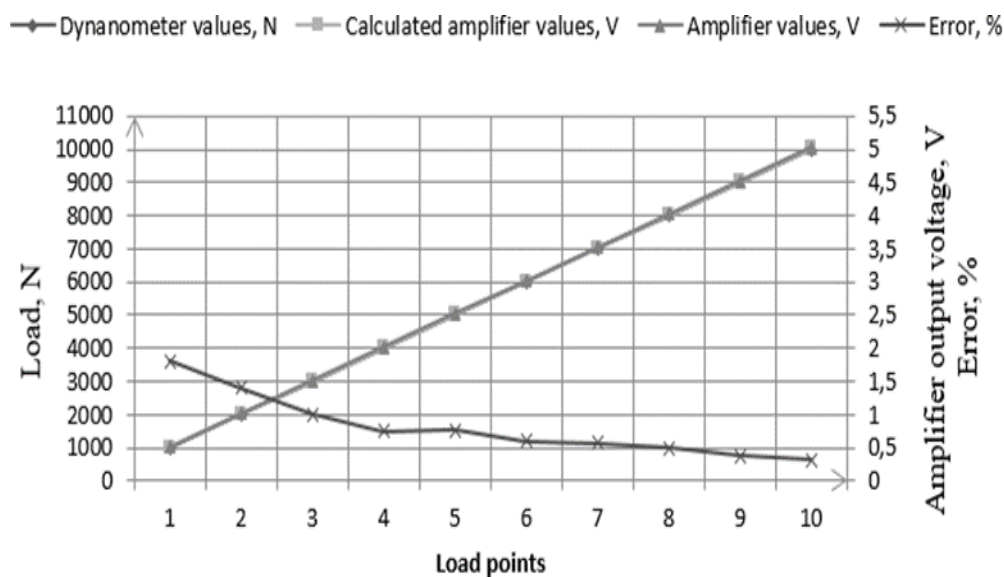


Figure 11: Results of Calibration

Experimental testing was performed in outdoor conditions. For thrust measurements static rocket motor testing stand was used with attached load cell. Amplifier and data recording computer was used at safe distance for motor igniter initiation. Fig. 12,0 shows stop frame of the particular experiment



Figure 12: Experimental testing

After experiment raw data was processed using Mat lab software which is shown in Fig.13.0 to extract only valuable information for obtaining thrust graph (Taylor et.al. 2019). One has found that modelled rocket and experimental rocket motor thrust graphs deviation was as predicted. One must be noted that there were some differences from compared modelled and experimental obtained graphs. In model graph igniter produced thrust force was not included into account which enchanted deviation at very beginning after thrust curve started to build up. From experimental data one has found out that rocket motor produced maximum thrust of 13054N and average thrust was 10576N before sliver. Total impulse was 32995 N·s and specific impulse was 186.6 second.

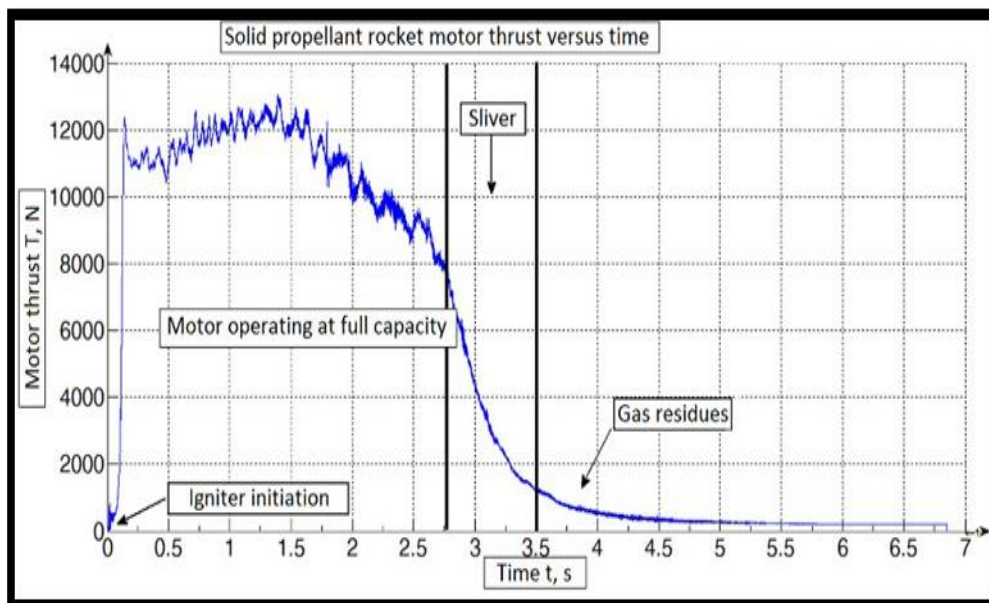


Figure 13: Thrust graph of rocket motor during experiment

VI. CONCLUSIONS

To test design requirements, rocket motor internal ballistics model was implemented with Matlab software. The maximum thrust of the modelled design was 12672 N, total impulse was 33000 N·s and specific impulse was 187 seconds. The thrust graph from the model was used for future design steps. Mechanical parameters were selected for most suitable thrust measurements. One was found that best design for load cell working point's is 10kN and 15kN. The bending strain of the load cell was 1547×10^{-6} and maximum bend of strain gage was 0.154% or 0.278°. The parametric model for mechanical geometry of the load cell was calculated and based on obtained data instrumental amplifier was designed. The theoretical model and real amplifier maximum error after comparison was 1.8 % at 1 kN load where output voltage was 0.509 V and minimum error of 0.319% at 10 kN load where output voltage was 5.019 V. After experimental-testing the measured rocket motor thrust characteristics was obtained. Maximum thrust which was produced from rocket motor was 13054N, total impulse was 32995N·s and specific impulse was 186.6 seconds. The error of modeled maximum thrust and measured maximum thrust was 2.92 %. The total impulse error was 0.015% and specific impulse error was 0.213%. After modeled and experimental results comparison one can consider the robustness and accuracy of the design method for thrust measurement system for solid propellant rocket motor.

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